

Portfolio Credit Risk Modelling and CDO Pricing - Analytics and Implied Trees from CDO Tranches

A Thesis Submitted for the Degree of
Doctor of Philosophy

by

Tao Peng

B.Sc.(Southwest University of Finance and Economics, China)

M.Sc. (University of Queensland)

`tao.peng-1@student.uts.edu.au`

in

School of Finance and Economics

University of Technology, Sydney

PO Box 123 Broadway

NSW 2007, Australia

May 10, 2010

Certificate

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Candidate

Date

Acknowledgements

I would like to thank my supervisor, Professor Erik Schlögl for his guidance, advice and patience through my PhD studies. His rigorous approach to academic research and the principles of research I learnt from him are critical for my academic career.

I have also benefited greatly from working with Peter Cotton at Julius Finance in New York and am grateful for his comments on the models and for sharing his insight on credit modeling. I would also like to give special thanks to Richard Stone at ANZ for his help on data collection and his knowledge on model testing.

To all of the staff at the School of Finance and Economics, I would like to take the opportunity to thank you for all your support and assistance. My research experience was enriched from participation and networking at the Quantitative Methods in Finance conference and the workshops organized by the Quantitative Finance Research Centre. Additionally, I learnt substantially from the master classes and seminars organized by the Financial Integrity Research Network where I had the opportunity to discuss with world renowned scholars in credit risk and relevant areas in quantitative finance. In particular the financial assistance for my research and traveling from both the school and the centre, the research scholarship I received from ARC and the generous resources provided with the help of Dzung Le at ANZ are also greatly appreciated.

Finally I would like thank my parents and my sister who have provided me for many years with endless support and continual encouragement.

Contents

Abstract	v
Chapter 1. Introduction	1
1.1. Literature Review	1
1.2. Motivation	40
1.3. Thesis Structure	40
Chapter 2. Tranche sensitivities and model design	42
2.1. Assumptions and results	44
2.2. Gaussian copula tranche sensitivities	45
2.3. Normal Inverse Gaussian copula tranche sensitivities	52
2.4. Spreads sensitivities to model parameters	63
2.5. Calibration algorithm for NIG copula and results	65
2.6. Dependence structure, model design and risk management	67
2.7. Conclusion	69
Chapter 3. Semi-parametric lattice models - a static binomial model	76
3.1. The features of the three models	76
3.2. A static binomial model	78
3.3. Model setup	78
3.4. Algorithmic construction of the model	80
3.5. Model calibration with CE method	82
3.6. Comparison with implied copula model by Hull and White	93
3.7. Conclusion	94
Chapter 4. A dynamic binomial model	95
4.1. The stochastic intensity process	95
4.2. The setup of a dynamic binomial lattice model	96
4.3. Algorithmic construction of conditional survival probabilities	97

4.4.	Calibration to marginal default probabilities	100
4.5.	Pricing on the path	101
4.6.	Calibration performance with the CE method	104
4.7.	Calibration results	105
4.8.	Implied market process	106
4.9.	Stochastic recovery rate extension for the binomial model	107
4.10.	Conclusion	116
Chapter 5.	The Markovian binomial model	117
5.1.	In search of a Markovian binomial model	117
5.2.	Model setup	118
5.3.	Calibration to marginal default probabilities on the nodes	121
5.4.	Modified calibration algorithm	126
5.5.	Calibration Results	126
5.6.	Model simplification	133
5.7.	Extension to stochastic recovery rate	134
5.8.	Comparison to the simplified approach by Hull and White	139
5.9.	Conclusion	140
Chapter 6.	Thesis Conclusion	152
	Bibliography	154

Abstract

One of the most successful and most controversial innovative financial products in recent years has been collateralised debt obligations (CDOs). The dimensionality of dependency embedded in a typical CDO structure poses great challenges for researchers - in both generating realistic default dynamics and correlation, and in the mean time achieving fast and accurate model calibration.

The research presented in this thesis contributes to the class of *bottom-up* models, which, as opposed to *top-down* models, start by modelling the individual obligor default process and then moving them up through the dependency structures to build up the loss distributions at the portfolio level.

The Gaussian model (Li 2000) is a *static* copula model. It has only one correlation parameter, which can be calibrated to one CDO tranche at a time. Its simplicity achieves wide spread industry application even though it suffers from the problem of 'correlation smile'. In other words, it cannot fit the market in an arbitrage-free manner in the capital-structure dimension.

The first contribution of this thesis is the sensitivities analysis with regard to model parameters of expected losses of CDO tranches in the Gaussian and NIG copula models. The study provided substantial insight into the essence of the dependency structure. In addition, we apply the intensity approach to credit modelling in order to imply market distributions non-parametrically in the form of a binomial lattice.

Under the same framework, we developed a series of three models. The static binomial model can be calibrated to the CDS index tranches exactly, with one set of parameters. The model can be seen as a non-parametric copula model that is arbitrage free in the capital-structure dimension.

Static models are not suitable to price portfolio credit derivatives that are dynamic in nature. The static model can be naturally developed into a *dynamic* binomial model and satisfies no-arbitrage conditions in the time dimension. This setup, however, reduces model flexibility and calibration speed. The computational complexity comes from the non-Markovian character of the default process in the dynamic model.

Inspired by Mortensen (2006), in which the author defines the intensity integral as a conditioning variable, we modify the dynamic model into a *Markovian* model by modelling the intensity integral directly, which greatly reduces the computational time and increases model fit in calibration. We also show that, when stochastic recovery rates are involved, there is a third no-arbitrage condition for the expected loss process that needs to be built into the Markovian model. For all binomial models, we adopt a unique optimisation algorithm for model calibration - the Cross Entropy method. It is particularly advantageous in solving large-scale non-linear optimisation problems with multiple local extrema, as encountered in our model.

